

Derivation of the Range Equation in Logarithmic Terms

The attenuation in free space of an RF signal is given as:

$$\left(\frac{4\pi d}{\lambda}\right)^2$$

Where d is the distance in metres, and λ is the wavelength of the RF signal. This equation yields a simple ratio, so to convert it to a decibel value we take 10 times the log of it:

$$10 \times \text{Log}\left(\frac{4\pi d}{\lambda}\right)^2$$

or we can do the squaring by doubling the log value, which yields:

$$20 \times \text{Log}\left(\frac{4\pi d}{\lambda}\right) \quad \text{Equ[1].}$$

As we know, $\lambda = \frac{c}{f}$, where c = the speed of light, or 300×10^6 metres/second, and f is the frequency in

Hz. We can divide top and bottom of this equation by 10^9 , which changes it to: $\lambda = \frac{0.3}{f(\text{GHz})}$ Equ[2].

Since the equation [1] has λ as its denominator, we can substitute the equation [2] by multiplying the first by its reciprocal, which yields:

$$20 \times \text{Log}\left(4\pi \cdot d \cdot \frac{f(\text{GHz})}{0.3}\right)$$

Or, separating the terms:

$$20 \times \text{Log}\left[\left(\frac{4\pi}{0.3}\right) \cdot f(\text{GHz}) \cdot d\right]$$

Moving the multiplication to the logarithmic domain: -

$$20 \times \left[\text{Log}\left(\frac{4\pi}{0.3}\right) + \text{Log}(f(\text{GHz})) + \text{Log}(d)\right]$$

As the first term is a constant, it can be expanded, and the multiplication can be distributed over the remaining terms, which yields the final form:

$$\underline{\underline{32.44 + 20\text{Log}[f(\text{GHz})] + 20\text{Log}[d]}}$$

Where d is the distance in metres.